

The Addition and Resolution of Vectors: The Force Table [NOTEBOOK LAB]

Objectives:

After completing this lab, you will be able to:

1. Add a set of vectors graphically to find the resultant.
2. Add a set of vectors analytically to find the resultant.
3. Appreciate the differences between graphical and analytical methods of vector addition.

Introduction:

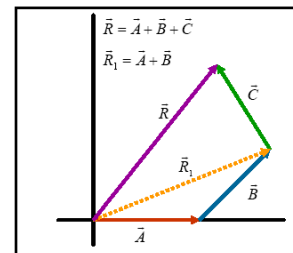
Physical quantities are generally classified as being scalar or vector quantities. The distinction is simple. A scalar quantity is one with a magnitude only for example, speed (*55 mph*) and time (*3 hrs*). A vector quantity on the other hand has both magnitude and direction. Such quantities include displacement, velocity, acceleration and force, for example, a velocity of *55 mph* north or a force of *20 N* along the *+y* axis.

Because vectors have the property of direction, the common method of addition, scalar addition, is not applicable to vector quantities. To find the resultant or vector sum of two or more vectors, special methods of vector addition are used, which may be graphical and/or analytical. Two of these methods will be described, and we will investigate the addition of force vectors. The result of graphical and analytical methods will be compared with the experimental results obtained from the force table. The experimental arrangement of forces (vectors) will physically illustrate the principles of the methods of vector addition.

Triangle (Head to Tail) Method:

Vectors are represented graphically by arrows. The length of a vector arrow (drawn to scale on graph paper) is proportional to the magnitude of the vector, and the arrow points in the direction of the vector.

The length scale is arbitrary and usually selected for convenience and so that the vector graph fits nicely on the graph paper. A typical scale for a force vector might be $1\text{ cm} = 10\text{ N}$. That is each centimeter of vector length represents ten newtons. The scale factor in this case in terms of force per unit length is 10 N/cm .



To add two vectors a triangle of which **A** and **B** are adjacent sides is formed. Vector arrows may be moved as long as they remain pointed in the same direction. The arrow that is the hypotenuse of the triangle is **R₁** (see figure above). This is the resultant or vector sum of **A + B**. The magnitude of **R₁** is proportional to the length of the diagonal arrow, and the direction of the resulting vector is that of the diagonal arrow **R₁**. The direction of **R₁** may be specified as being at an angle relative to the x-axis.

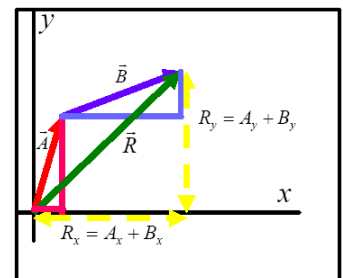
Polygon Method

If more than two vectors are added, the head-to-tail method forms a polygon (see figure above). For three vectors, the resultant equals **A + B + C** is the vector arrow from the tail of the vector **A** to the head of the vector **C**. The length (magnitude) and the angle of orientation of **R** can be measured from vector diagram using a ruler and a protractor.

Note that this equivalent to applying the head-to-tail method (two vectors) twice. **A** and **B** are added to give **R₁**, then **C** is added to **R₁** to give **R**.

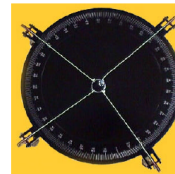
Component Method:

We may resolve any vector into x and y components. That is, a vector **R** is the resultant of **R_x** and **R_y**. The magnitude and the direction of **R** is given by the vector sum of any number of vectors that can be obtained by adding the x and y components of the vectors. The magnitude of the resultant is given by summing the x components and then summing the y components and then finally using the Pythagorean Theorem where the x and y are the legs of the triangle and the resultant is the hypotenuse.



In other words:

We may resolve any vector into x and y components. That is, a vector \vec{R} is the resultant of R_x and R_y , and $\vec{R} = \vec{R}_x + \vec{R}_y$, where $R_x = R \cos \theta$ and $R_y = R \sin \theta$. The magnitude and the direction of \vec{R} is given by



$$R = \sqrt{R_x^2 + R_y^2}$$
$$\theta = \tan^{-1} \left[\frac{R_y}{R_x} \right]$$

The vector sum of any number of vectors can be obtained by adding the x and y components of the vectors. The magnitude of the resultant is given by $R = \sqrt{R_x^2 + R_y^2}$,

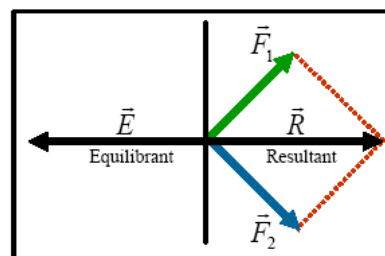
where $R_x = A_x + B_x + C_x + \dots$ and $R_y = A_y + B_y + C_y + \dots$ and $\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$.

Force Tables:

The force table is an apparatus that allows the experimental determination of the resultant of force vectors. The rim of the circular table is calibrated in degrees. Weight forces are applied to a central ring by means of strings running over pulleys and attached to weight hangers. The magnitude (which is the mass times the acceleration due to gravity) of a force (vector) is varied by adding or removing slotted weights, and direction is varied by moving the pulley.

The resultant of two or more forces (vectors) is found by balancing the forces with another force (weights on a hanger) so that the ring is centered on the center pin. The balancing force is not the resulting R , but rather the equilibrant E , or the force that balances the other forces and holds the ring in the equilibrium.

The equilibrant is the vector force of equal magnitude, but in the opposite direction, to the resultant like the diagram below.



Apparatus:

- | | |
|-----------------------|----------------------|
| 1. Vector Force table | 2. Masses on hangers |
| 3. Graph paper | 4. Ruler |
| 5. Protractor | 6. String |

Procedure

There will be two trials for this lab. Follow this procedure for each trial below.

1. Create your Lab Notebook Entry. All information will be entered into the notebook.
2. After attaching the pulleys and mass hooks to the table, arrange the three forces using the list above for trial 1.
3. Determine experimentally the fourth force (the equilibrant) required to balance the ring in the center of the table. Remember, this fourth force is equal in magnitude, but opposite in direction, to the resultant of the three forces. Record the **resultant** in the Data Table under Equilibrium.
4. Take your equilibrium mass off of the hook.
5. Find the resultant of the three forces *graphically* using methods learned in class. Show your scale, and record your result in the Data Table.
6. Find the resultant of the three forces *analytically* using methods learned in class. Show your work, and record your result in the Data Table.
7. Repeat for Trial 2. (remember units in the table!)
8. Pick 3 random masses and angles and find the equilibrium for your three forces.

Data:

| Force Table Data | | | | | | | | |
|------------------|---------|-----------------|---------|-----------------|---------|-----------------|-------------|-------------------|
| Experiment | Mass #1 | Mass #1's Angle | Mass #2 | Mass #2's Angle | Mass #3 | Mass #3's Angle | Equilibrium | Equilibrium Angle |
| 1 | 75g | 25° | 105g | 75° | 225g | 150° | | |
| 2 | 55g | 45° | 125g | 300° | 65g | 210° | | |
| 3 | | | | | | | | |

| Vector Addition Results | | | |
|---------------------------|--|---|---|
| | Hands-on Force Table (from Table Average) (Experimental Value) | GRAPHICAL (from drawing) (Experimental Value) | ANALYTICAL (from equations) (Theoretical Value) |
| MAGNITUDE TRIAL #1 | | | |
| DIRECTION TRIAL #1 | | | |
| MAGNITUDE TRIAL #2 | | | |
| DIRECTION TRIAL #2 | | | |
| MAGNITUDE TRIAL #3 | | | |
| DIRECTION TRIAL #3 | | | |

*Compare the Percent Error's of the Magnitudes Only.

| Percent Error of Results | | |
|------------------------------|-----------------------------------|--------------------------|
| | Hands-on Force Table (from Table) | GRAPHICAL (from drawing) |
| MAGNITUDE TRIAL #1 | | |
| MAGNITUDE TRIAL #2 | | |
| MAGNITUDE TRIAL #3 | | |
| AVERAGE PERCENT ERROR | | |

Questions/Things you need to do individually:

Purpose (3pt)

Include

Data (10pt)

Print and attach the data tables into your notebook.

Graphs (10pt)

Print and attach the data tables into your notebook.

Calculations (10pt)

Show your calculations and drawings for all 3 trials

Error Analysis (10pt)

Calculate the percent error for your graphs magnitude compared to the analytical values magnitude. You only have one data trial so you cannot calculate Average Deviation from the Mean nor can you calculate Average Deviation of the Mean.

Questions (8pt)

Answer the following questions:

1. What is the largest possible resultant vector from two vectors of lengths 3 cm and 4 cm? What is the smallest possible resultant?
2. Two vectors are originally parallel. How does the resultant change as the angle between the two increases to 180° ?
3. Could you ever have a resultant vector *shorter* than one of its components? Could you ever have a resultant vector *equal* to one of its components? Explain why.
4. Could all four pulleys be placed in the same quadrant or in two adjacent quadrants and still be in equilibrium? Explain why.

Conclusions (4pt)

A normal conclusion.