Rotational Dynamics
Smart Pulley

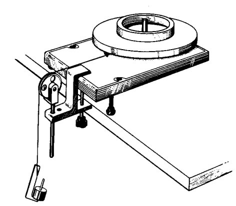
The motion of the flywheel of a steam engine, an airplane propeller, and any rotating wheel are examples of a very important type of motion called rotational motion. If a rigid body is acted upon by a system of torques, the body will be in equilibrium, as far as rotational motion is concerned, if the sum of the torques about any axis is zero. This means that if the body is at rest, it will remain at rest; if it is rotating about a fixed axis, it will continue to rotate about the same axis with uniform angular speed. However, if an unbalanced torque is acting on the body, it will produce an angular acceleration in the direction of the torque, the acceleration being proportional to the torque and inversely proportional to the moment of inertia of the body about its axis of rotation. The purpose of this experiment is to study rotational motion, to observe the effect of a constant torque upon a body that is free to rotate, and to determine the resulting angular acceleration and the moment of inertia of that body.

THEORY

When a torque is applied to a body free to rotate about a fixed axis, then the body will acquire an angular acceleration given by the relation **Figure 2** *Forces acting on a falling rnass*

$$\tau = I\alpha \qquad (1)$$

where τ is the sum in meter-newtons of all the torques about the fixed axis of rotation, I is the moment of inertia in kilogram-meters^2 of the body about the same axis, and a is the angular acceleration in radians per second per second.



The moment of inertia is the inertia of a body as regards rotation about an axis. The moment of inertia of a body with respect to an axis is the sum of the products obtained by multiplying the mass of every particle of the body by the square of its distance from the axis. For a simple geometrical solid it can be readily determined by the use of integral calculus. For example, the moment of inertia of a uniform cylinder, or disk, with respect to its longitudinal axis is given by

$$I = \frac{1}{2}MR^2 \qquad (2)$$

where M is the mass of the cylinder or disk; R is its radius in meters; and I is the moment of inertia in kilogram-meters^2
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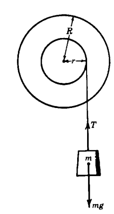

Figure 2 *Forces acting on a falling mass*

The apparatus used in this experiment is shown in Fig. 1. It consists of a disk mounted on a shaft which is carried in a very good bearing in the base plate so that the disk is free to rotate with a minimum of friction. On its underside (and thus not visible in the figure), the disk carries a drum with three diameters around any one of which a string may be wound so as to exert a torque on the rotating assembly. The string passes over a pulley and has a weight attached to the end. When this weight is released, it descends with a constant linear acceleration, while the rotating assembly turns with a constant angular acceleration. This situation is illustrated in a simplified form in Fig. 2. Note that in the actual apparatus the disk is mounted in a horizontal plane so that a second disk, a ring, and a bar may be loaded onto it to vary the moment of inertia of the complete assembly. A pulley is thus required to change the string direction from horizontal to vertical. This pulley is chosen to be very light and small so as to contribute a negligible moment of inertia, and its contribution to the friction in the apparatus will be included with the friction in the bearing supporting the disk. As we will neglect the effect of the pulley, Fig. 2 is drawn as if the disk of radius R were in a vertical plane so that the string, which is wound around a drum of radius r, can be shown going straight down to the falling mass m. We see that there are two forces acting on this mass: the downward force of gravity, that is, the mass's weight *mg*, and the upward pull of the string, namely its tension T.

CALCULATIONS

1. ***Torque and acceleration analysis*** (consider the mass of the pulley to be negligibly small): Let us examine the forces that cause the hanging mass to accelerate downward and the torque that causes the spinning part of the apparatus to acquire a rotational acceleration. There are two forces that act on the hanging mass: its weight (force of the earth on m) **mg**, and the force of the string, **FT** (tension in the string). Applying Newton's Second Law we can write:

$$\mathbf{mg - F_T = ma} \quad \text{(Eqn. 1)}$$

For the rotational apparatus we need to set the total torque (about the rotational axis) equal to the rotational inertia of the apparatus, **I**, multiplied by its angular acceleration, α. Since the only force that produces a non-zero torque about this axis is due to the tension in the string, we can write: α (Eqn. 2)

.

$$\tau_{t\,=}\ \mathbf{I}\ \boldsymbol{\alpha} \quad \text{(Eqn. 2)}$$

But the angular acceleration α is related to the tangential acceleration, **a**, at the rim of the spool (i.e. the acceleration of the string) by:

$$\mathbf{a = r}\ \alpha \quad \text{(Eqn. 3)}$$

Also, by using the definition of torque, we can write:

$$\tau_t = \mathbf{FT\ r} \quad \text{(Eqn. 4)}$$

By substituting Eqns. 3 and 4 in 2 we obtain:

$$\mathbf{F_T\ r = I\ a / r} \quad \text{(Eqn. 5)}$$

Finally, we can combine Eqns. 5 and 1 to obtain an expression for the rotational inertia of the system in terms of all the other variables:

$$\mathbf{I = m\ [(g/a)\text{-}1]\ r^2} \quad \text{(Eqn. 6)}$$

NOTE:
r= the radius of the small disk under the main disk
a = the final acceleration of the hanging mass

So, by measuring the acceleration of the falling mass we could determine the rotational inertia of the system.

2. ***Energy Considerations***:
Consider when the mass **m** is released and its speed **v** is determined exactly at the point where it has fallen a height **h**. Assuming negligible frictional forces we can set the loss in the potential energy of the falling mass equal to the gain in its translational kinetic energy plus the gain in the rotational kinetic energy of the system, i.e.:

$$\mathbf{mgh = ½\ mv2 + ½\ I\omega 2} \quad \text{(Eqn. 6).}$$

Now, the angular speed of the rotating apparatus is related to the tangential speed at the rim of the spool (i.e. the speed of the string **-** i.e. the speed of the falling mass) according to:

$$\mathbf{v = r}\ \omega \quad \text{(Eqn. 7)}$$

Substituting Eqn. 7 into 6 we obtain an expression for the rotational inertia of the system as:

$$\mathbf{I = [(2gh/\ v2) - 1]\ m\ r2} \quad \text{(Eqn. 8).}$$

APPARATUS

1. Rotational motion apparatus, consisting of a main disk and drum, base plate with bearing and shaft, auxiliary disk, steel ring, and rectangular plate
2. smart pulley and table edge clamp
3. Science Workshop interface
4. Weight hanger with 50- and 100-g weights
5. Stopwatch or stop clock
6. Meter stick
7. Vernier calipers
8. Triple-beam balance
9. Level
10. String

PROCEDURE

Note: Creating the tables in Excel with the correct formulas will speed up your calculations and data collection

Calculated Moment of inertia I for the main disk, auxiliary disk and the ring.

1. Measure the mass of the main disk and the radius. Record.
2. Calculate the moment of inertia for the main disk based on $\mathbf{I=1/2MR^2}$ record your answers in table 1
3. Calculate the moment of inertia for the auxiliary disk based on $\mathbf{I=1/2MR^2}$ record your answers in table 1
4. Calculate the moment of inertia for the ring based on $\mathbf{I=MR^2}$ record your answers in table 1
5. Record the moment of inertia for the main disk in table 3
6. Calculate the moment of inertia for main disk and auxiliary disk by adding together the moment of inertia of the auxiliary and main disk record your answers in table 3
7. Calculate the moment of inertia for main disk and ring by adding together the main disk and the ring record your answers in table 3

Experimental moment of inertia

8. Set up data studio as follows
   - Open
   - Click smart Pulley
   - **Select position ch 1 and velocity ch1**
   - Click on constants
   - If you have a 3 spoke pulley Set arc length at .05 and spoke angle 120^0 for the three spoke smart pulley. If you have the black 10 spoke pulley do not change the settings.
   - To set auto stop, click sampling options
   - click auto stop
   - click data measurement
   - change from velocity ch 1 to position ch 1
   - change to is above
   - set .28 m and click ok.
   - Set up velocity time graph as follows: (Remember the slope is the acceleration)
     a. Under displays click on graph
     b. Click on velocity ch 1
     c. Click anywhere on graph to get graph setting
     d. Now click on axis settings
     e. Set both x and y minimum at 0 and x maximum at 4 seconds
     f. You are now ready to start the experiment

9. Wind up the string on the large step pulley under the main disk.
10. Place 100 g mass on mass hanger.
11. Make sure the step pulley is alignment with the smart pulley by making sure the string runs horizontal.
12. Place the string over the smart pulley and then hold the disk to prevent the string from unwinding.
13. Start program and release disk so pulley will unwind.
14. The autostop will turn of the program
15. The slope of the velocity time graph you get will be the tangential acceleration of the disk.
16. To get the slope of the graph click on fit and then linear. This will displace the slope of the line. Record the Acceleration in table 2.

17. Repeat 4 more times. Take the average.
18. Repeat step 16 and 17 using the main disk and the auxiliary disk.
19. Repeat step 16 and 17 using the main disk and the hoop.
20. This lab will not be done using the computer. It will be done on notebook paper long hand.
21. Calculate the force on the step pulley as follows and record it in table 3.
    - Use Equation 1 based on Newton's second law,

$$ma = F_{applied} - F_{tension}$$

$F_{applied}$ is the weight on the pulley (mg)
$F_{tension}$ is the force on the step pulley. The force we use to calculate torque.

Rearrange the above equation.

$$\mathbf{F_{tension} = mg - ma.}$$

m is the hanging mass. Include the mass of the mass hanger (5 g) if present.
a is the acceleration you measured.

22. Calculate the torque on the pulley and record it in table 3.

$$\boldsymbol{\tau} \textbf{ = force x distance}$$

$$\text{Force} = F_{tension}$$

distance = radius of the step pulley = .025m.
Measure it with vernier calipers to verify this value.

23. Calculate angular acceleration (α) record in table 3.

$$a = \alpha r \quad \text{so} \quad \boldsymbol{\alpha} = \mathbf{a/r}$$

r of the step pulley

24. Calculate the moment of inertia record in table 3.

$$\tau = I\alpha \quad \mathbf{I = \boldsymbol{\tau}/\boldsymbol{\alpha}}$$

This is the experimental I.

25. Calculate the percent error between the calculated moment of inertial and the experimental moment of inertial Record in table 4

DATA TABLES

TABLE 1 (Calculated Moment of Inertia)			
Item	**mass (kg)**	**radius (m)**	**Calculated Moment of Inertia ($kg\ m^2$)**
Main disk			
auxiliary disk			
ring			

Table 2 (Accelerations)			
Trial	**Main disk (m/s^2)**	**Main disk + auxiliary (m/s^2)**	**Main disk + ring (m/s^2)**
1			
2			
3			
4			
5			
Average=			

Table 3 (Experimental Moment of Inertia)						
Item	**Calculated Moment of Inertia ($kg\ m^2$)**	**Acceleration from slope (m/s^2)**	**Tension F_t (N)**	**Torque τ (Nm)**	**Angular Acceleration α (rad/s^2)**	**Experimental Moment of Inertia ($kg\ m^2$)**
Main disk						
main +auxiliary						
main + ring						

Table 4 (Errors)			
Item	**Calculated Moment of Inertia ($kg\ m^2$)**	**Experimental Moment of Inertia ($kg\ m^2$)**	**error (%)**
Main disk			
main +auxiliary			
main + ring			