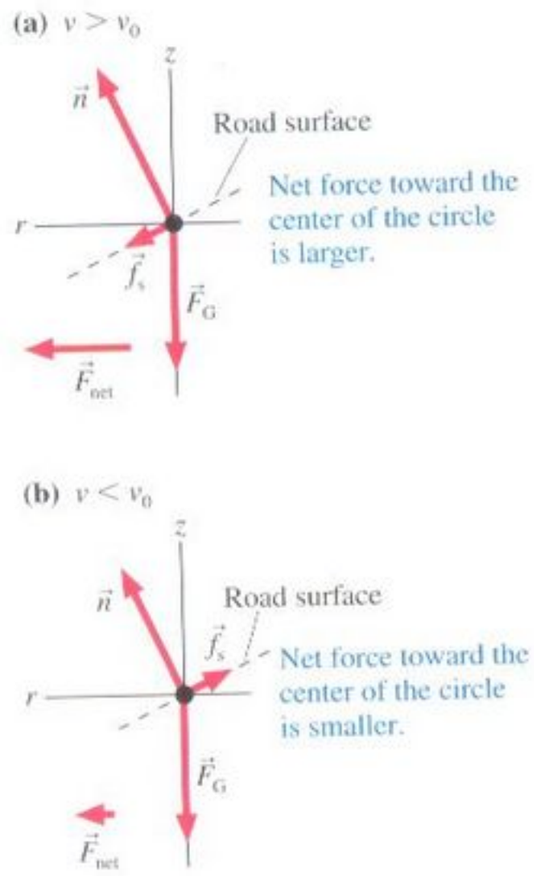
FIGURE 8.10 Free-body diagrams showing the static friction force when $v > v_0$ and when $v < v_0$.



It's interesting to explore what happens at other speeds. The car will need to rely on both the banking *and* friction if it takes the curve at a speed higher or lower than v_0. **FIGURE 8.10a** has modified the free-body diagram to include a static friction force. Remember that $\vec{f}_s$ must be parallel to the surface, so it is tilted downward at angle θ. Because $\vec{f}_s$ has a component in the positive r-direction, the *net* radial force is larger than that provided by $\vec{n}$ alone. This will allow the car to take the curve at $v > v_0$. We could use a quantitative analysis similar to Example 8.5 to determine the maximum speed on a banked curve by analyzing Figure 8.10a when $f_s = f_{s\,max}$.

But what about taking the curve at a speed $v < v_0$? In this situation, the r-component of the normal force is too big; not that much center-directed force is needed. As **FIGURE 8.10b** shows, the net force can be reduced by having $\vec{f}_s$ point *up* the slope! This seems very strange at first, but consider the limiting case in which the car is parked on the banked curve, with $v = 0$. Were it not for a static friction force pointing *up* the slope, the car would slide sideways down the incline. In fact, for any speed less than v_0 the car will slip to the inside of the curve unless it is prevented from doing so by a static friction force pointing up the slope.

Our analysis thus finds three divisions of speed. At v_0, the car turns the corner with no assistance from friction. At greater speeds, the car will slide out of the curve unless an inward-directed friction force increases the size of the net force. And last, at lesser speeds, the car will slip down the incline unless an outward-directed friction force prevents it from doing so.

EXAMPLE 8.6 A rock in a sling

A Stone Age hunter places a 1.0 kg rock in a sling and swings it in a horizontal circle around his head on a 1.0-m-long vine. If the vine breaks at a tension of 200 N, what is the maximum angular speed, in rpm, with which he can swing the rock?

MODEL Model the rock as a particle in uniform circular motion.

VISUALIZE This problem appears, at first, to be essentially the same as Example 8.3, where the father spun his child around on a rope. However, the lack of a normal force from a supporting surface makes a *big* difference. In this case, the *only* contact force on the rock is the tension in the vine. Because the rock moves in a horizontal circle, you may be tempted to draw a free-body diagram like **FIGURE 8.11a**, where $\vec{T}$ is directed along the r-axis. You will quickly run into trouble, however, because this diagram has a net force in the z-direction and it is impossible to satisfy $\sum F_z = 0$. The gravitational force $\vec{F}_G$ certainly points vertically downward, so the difficulty must be with $\vec{T}$.

As an experiment, tie a small weight to a string, swing it over your head, and check the *angle* of the string. You will quickly discover that the string is *not* horizontal but, instead, is angled downward. The sketch of **FIGURE 8.11b** labels the angle θ. Notice that the rock moves in a *horizontal* circle, so the center of the circle is *not* at his hand. The r-axis points to the center of the circle, but the tension force is directed along the vine. Thus the correct free-body diagram is the one in Figure 8.11b.

SOLVE The free-body diagram shows that the downward gravitational force is balanced by an upward component of the tension, leaving the radial component of the tension to cause the centripetal acceleration. Newton's second law is

$$\sum F_r = T\cos\theta = \frac{mv^2}{r}$$

$$\sum F_z = T\sin\theta - mg = 0$$

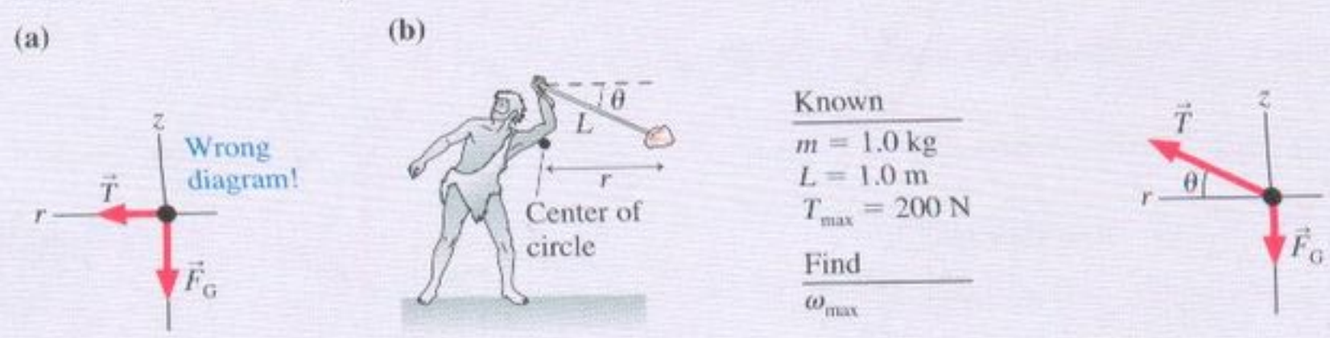
where θ is the angle of the vine below horizontal. From the z-equation we find

$$\sin\theta = \frac{mg}{T}$$

$$\theta = \sin^{-1}\left(\frac{(1.0\text{ kg})(9.8\text{ m/s}^2)}{200\text{ N}}\right) = 2.81°$$

where we've evaluated the angle at the maximum tension of 200 N. The vine's angle of inclination is small but not zero.

FIGURE 8.11 Pictorial representation of a rock in a sling.



Turning now to the r-equation, we find the rock's speed is

$$v = \sqrt{\frac{rT\cos\theta}{m}}$$

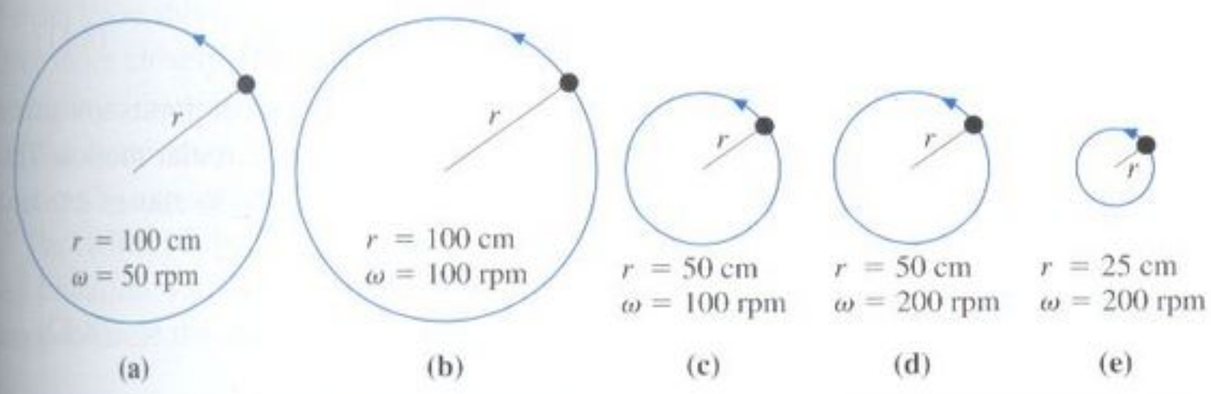
Careful! The radius r of the circle is *not* the length L of the vine. You can see in Figure 8.11b that $r = L\cos\theta$. Thus

$$v = \sqrt{\frac{LT\cos^2\theta}{m}} = \sqrt{\frac{(1.0\text{ m})(200\text{ N})(\cos 2.81°)^2}{1.0\text{ kg}}} = 14.1\text{ m/s}$$

We can now find the maximum angular speed, the value of ω that brings the tension to the breaking point:

$$\omega_{max} = \frac{v}{r} = \frac{v}{L\cos\theta} = \frac{14.1\text{ rad}}{1\text{ s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 135\text{ rpm}$$

STOP TO THINK 8.3 A block on a string spins in a horizontal circle on a frictionless table. Rank order, from largest to smallest, the tensions T_a to T_e acting on blocks a to e.



8.4 Circular Orbits

Satellites orbit the earth, the earth orbits the sun, and our entire solar system orbits the center of the Milky Way galaxy. Not all orbits are circular, but in this section we'll limit our analysis to circular orbits. We'll look at the elliptical orbits of satellites and planets in Chapter 13.

How does a satellite orbit the earth? What forces act on it? Why does it move in a circle? To answer these important questions, let's return, for a moment, to projectile motion. Projectile motion occurs when the only force on an object is gravity. Our analysis of projectiles assumed that the earth is flat and that the acceleration due to gravity is everywhere straight down. This is an acceptable approximation for projectiles of limited range, such as baseballs or cannon balls, but there comes a point where we can no longer ignore the curvature of the earth.

FIGURE 8.12 shows a perfectly smooth, spherical, airless planet with one tower of height h. A projectile is launched from this tower parallel to the ground ($\theta = 0°$) with speed v_0. If v_0 is very small, as in trajectory A, the "flat-earth approximation" is valid and the problem is identical to Example 4.4 in which a car drove off a cliff. The projectile simply falls to the ground along a parabolic trajectory.

As the initial speed v_0 is increased, the projectile begins to notice that the ground is curving out from beneath it. It is falling the entire time, always getting closer to the ground, but the distance that the projectile travels before finally reaching the ground—that is, its range—increases because the projectile must "catch up" with the ground that is curving away from it. Trajectories B and C are of this type. The actual calculation of these trajectories is beyond the scope of this textbook, but you should be able to understand the factors that influence the trajectory.

If the launch speed v_0 is sufficiently large, there comes a point where the curve of the trajectory and the curve of the earth are parallel. In this case, the projectile "falls" but it never gets any closer to the ground! This is the situation for trajectory D. A closed trajectory around a planet or star, such as trajectory D, is called an **orbit.**

FIGURE 8.12 Projectiles being launched at increasing speeds from height h on a smooth, airless planet.